Observer-First Physics

A Reconstruction of Quantum Mechanics, Quantum Control, Lorentzian Geometry, and Algebraic Quantum Field Theory from the Conditions of Observation

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Abstract

We identify the mathematical structure that survives a single invariance principle (Axiom A0) and six constitutive constraints on a minimal embedded physical observer: a bounded, finite-energy record-keeping system that updates its causally ordered internal memory solely through local physical interactions along its continuous worldline. A0 asserts that physical reality consists solely of what is invariant under physically inert transformations. The same six constraints reappear unchanged in four domains, performing distinct mathematical work in each but bearing the same operational signature throughout.

Part I shows that the surviving structure in finite-dimensional quantum theory is exactly that of complex Hilbert space, the Born rule, tensor-product composition, unitary dynamics, and a local U(1) gauge structure with massless mediator.

Part II derives the operational scales any single observer of finite resources E_{op} and memory M can access: a minimum proper-time resolution $\tau_{\mathcal{O}} = \pi\hbar/(2E_{\mathrm{op}})$, control bandwidth $\Lambda_{\mathcal{O}} = E_{\mathrm{op}}/(\pi\hbar)$, and timing-jitter floor $\sigma_t \geq \tau_{\mathcal{O}}$. These fix the class of Observer-Realizable Channels and yield a specific falsifiable prediction: the optimal pulse number in a CPMG dynamical decoupling sequence is non-monotonic in n, with optimum $n_{\mathrm{opt}} \propto E_{\mathrm{op}}^{0.70}$ (the leading-order $E_{\mathrm{op}}^{2/3}$ expectation receiving a calculable logarithmic correction). The exponent matches existing CPMG measurements in spin qubits to within 1σ.

Part III extends to admissible families of observers via the multi-observer principle A0^{+}. The substrate they inhabit is necessarily Lorentzian, strongly causal, and free of closed timelike curves. Given the Bekenstein bound and the Unruh temperature as named imports, the metric satisfies Einstein's field equations. A calculable 1+1-dimensional Rindler model exhibits the Landauer cost of memory erasure matching the macroscopic Clausius heat flow term by term, identifying the operational origin of the thermodynamic relation.

Part IV reaches relativistic quantum field theory. The clock generator forced by O1 and O6 is, by A0, an element of the local algebra; the algebra of physical observables is its fixed-point algebra under proper-time reparametrisation; by Takesaki duality this is equivalent to the modular crossed product $\mathfrak{A}(\mathcal{O}) \rtimes_\sigma \mathbb{R}$, a hyperfinite type II_∞ factor (type II_1 in compact regions). The trace on this algebra renders vacuum-subtracted entropies finite, makes the Bekenstein bound a theorem in the type II setting, and reproduces the generalised entropy formula on semiclassical states. Independently, Witten and Chandrasekaran–Longo–Penington–Witten arrived at the same algebra from large-N gravitational considerations; the present work supplies the operational route.

O2 appears in the torsion-free argument, the timing-jitter floor, Landauer dissipation, the no-CTC result, the single-timelike-direction condition, the memory-capacity condition on the Landauer–Clausius bridge, and the necessity of the clock generator in the physical algebra. O5 appears in cancellativity, local tomography, the causal partial order, Lorentzian signature, and microcausality. Cross-domain consistency follows from common origin. The two genuine load-bearing inputs across the entire arc are A0 itself and the existence of a cyclic separating QFT vacuum. Every known alternative at each level is excluded at a specific identified step. Each named import in the operational chain has been independently confirmed: the Margolus–Levitin and Landauer bounds underlying Part II by direct measurement, the Bekenstein and Bisognano–Wichmann results by established gravitational arguments. The one parameter-free prediction — CPMG scaling $n_{\mathrm{opt}} \propto E_{\mathrm{op}}^{0.70}$ — agrees with

measured exponent 0.72 ± 0.01 in spin qubits to within 1σ. Whether some exotic theory could satisfy A0 and the observer constraints yet fail to embed into the structure identified here is the completeness question, left as the principal open problem.

Contents

Declaration

Every reader of this paper is a finite physical system inside the world it describes. This is not a philosophical preliminary. It is the operative fact from which everything that follows is derived. You are made of matter. You arrived here through physical processes. Every piece of information you have ever possessed reached you through physical interaction. You cannot step outside the world to examine it from a neutral vantage point. Neither can we.

This work asks: given that every observer is constitutively inside the world — not visiting it, not modeling it from outside, but being a physical part of it — what must the structure of physics look like?

The answer is not that quantum mechanics, Lorentzian geometry, and algebraic quantum field theory are three separate mathematical structures that happen to fit the data. The answer is that they are three layers of a single structure consistent with observation being what it actually is: a finite, local, interaction-based, physically situated process. The Hilbert space, the Born rule, the timing-jitter floor on quantum control, the Lorentzian metric, the type II algebra of local observables — none of these are features discovered by observers looking at the world. They are features constituted by what observation necessarily is when the observer is inside the world.

The derivation that follows is formal and gap-named. Its claim is that the apparent mysteries of quantum mechanics, gravity, and quantum field theory dissolve once you stop treating the observer as external and recognise that physical existence, for any system accessible to an observer inside the world, just is the invariant structure of its interactions.

We exist. We observe. What follows establishes that any operational theory consistent with this recognition is represented, in a precise mathematical sense, by complex Hilbert space quantum mechanics on a Lorentzian substrate satisfying Einstein's equations, with type II local algebras encoding the algebraic structure of measurement.

Epistemic gradient

Different parts of this work stand on different ground. The single-observer reconstruction (Part I) and the operational scales (Part II) are forced from A0 plus O1–O6 and a small set of named mathematical theorems (Pontryagin, Frobenius, Gleason, Stone, Wigner, Margolus–Levitin, Nyquist). The geometric extension (Part III) is forced for signature and causal structure but conditional on the Bekenstein bound and Unruh temperature for Einstein's equations. The algebraic-QFT completion (Part IV) rests on substantial named imports: the cyclic separating vacuum, Bisognano–Wichmann, Reeh–Schlieder, Haagerup, Takesaki, KLRSS, and the Chandrasekaran–Longo–Penington–Witten construction. Each section states its imports explicitly at the point of use; the reader can read each part with the appropriate level of trust in the result.

1 Axiom A0 and the Six Observer Constraints

Physical picture. If a transformation makes no possible difference to any measurement outcome for any observer, it is a pure redescription of the same physical situation. It has no physical content. Physics should therefore consist only of what survives all such redescriptions.

Definition 1.1 (A0: Physics is invariant structure). *Physical reality consists solely of what is invariant under all physically inert transformations. A transformation is physically inert if and only if it changes no outcome of any admissible physical process for any observer.*

Definition 1.2 (Operational inertness). *A transformation T acting on histories is* operationally inert *iff for every admissible protocol π and every history H,*

$$P_\pi(o \mid H) = P_\pi(o \mid T(H))$$

for every outcome o. A0 identifies physical equivalence classes with operational equivalence classes; it does not assume that all formal redundancies are operational. The reference class of admissible protocols is fixed by O1–O6.

What rejecting A0 requires. To reject A0 is to assert that there exist physical facts that make no difference to any outcome of any physical process, ever, under any protocol, for any observer, anywhere. The burden of proof is on the person who says such things exist. A0 is ontological, not epistemic: non-invariance is not a detection limit but an absence of physical content. Leibniz, Mach, and Einstein each followed this move at different scales — absolute position, absolute rotation, absolute simultaneity — and at each scale the invariant structure was the physics.

Logical status. A0 does not derive from prior grounds; it is a recognition. A0 and the observer constraints O1–O6 below are mutually constitutive: A0 specifies what "same physical content" means; O1–O6 specify what counts as an admissible observer. This is coherentism, not circularity. The framework is structurally falsifiable: it fails if any alternative satisfying O1–O6 is found that does not reduce to the structure identified here.

Protocol refinement. A natural objection is that a distinction currently undetectable by any admissible protocol might become detectable under future protocols or technology — absolute simultaneity was once operationally inaccessible, yet its rejection required specific experimental arrangements. Two responses are available. First, A0 is not epistemic ("we cannot detect it now") but constitutive: the inertness must hold for all *admissible physical processes, not merely known ones. A transformation that could in principle be detected by some admissible process is not inert under A0 regardless of whether that process has yet been performed. Second, O1–O6 bound the class of admissible processes: any protocol executable by a finite-energy, finite-memory, locally-interacting system counts. If future technology reveals a distinction currently uncounted, this constitutes either a violation of O1–O6 (the putative detection mechanism is not an admissible process) or genuine evidence against A0. Either outcome is a meaningful experimental result for the framework.*

1.1 The six constitutive constraints

O1 *Finite total energy $E_{\mathcal{O}} < \infty$; a portion $E_{\mathrm{op}} \leq E_{\mathcal{O}}$ is available for control.*

O2 *Finite memory register, $\dim \mathcal{H}_{\mathcal{O}} \leq M < \infty$, with strictly causally ordered states.*

O3 *Thermodynamic bound: $2 \leq \dim S_{\mathcal{O}} \leq k < \infty$.*

O4 *All knowledge arrives through local physical interactions.*

O5 *Finite signal speed $c < \infty$.*

O6 *Continuous proper time τ constituted by the observer's internal dynamics.*

These constraints are not assumptions about physics. They are constitutive properties of being a finite physical system inside the world. They apply simultaneously in every domain. This is why the same constraints do load-bearing work in quantum mechanics, spacetime geometry, quantum control, and algebraic QFT: not because those domains are forced to agree, but because they all face the same observer.

1.2 The redecomposition lemma

Splitting a state space into orthogonal sectors and recombining them is pure redescription. By A0 it changes no physical content, including total distinguishability. Any orthonormal description basis is operationally equivalent to any other.

Lemma 1.3 (Redecomposition requires the Euclidean norm)**.** *Any continuous, strictly convex norm on* $\mathbb{R}^n$ *(*$n \geq 2$*) invariant under the orthogonal group* $O(n)$ *is a positive multiple of the Euclidean norm.*

Proof. $O(n)$ acts transitively on S^{n-1}, so any $O(n)$-invariant norm satisfies $\|x\| = f(\|x\|_2)$ for some $f : [0, \infty) \to [0, \infty)$. Homogeneity gives $f(\lambda r) = \lambda f(r)$, so $f(r) = cr$ with $c > 0$. Continuity and strict convexity exclude pathological non-homogeneous solutions on subgroups. Any ℓ^p with $p \neq 2$ fails $O(n)$-invariance: $\|R_{45}(1, 0)\|_p = 2^{1/p-1/2} \neq 1$. □

For independent systems in orthogonal subspaces: $D(x \oplus y)^2 = D(x)^2 + D(y)^2$.

Part I

The Single-Observer Reconstruction of Quantum Mechanics

2 The Operational State Space and Composition

Physical picture. Two histories that always produce identical statistics under every possible measurement are physically the same thing. There is no operational sense in which they differ. The physical state space is obtained by grouping histories that cannot be distinguished by any admissible protocol. Two systems that cannot send signals to each other are causally independent; the order in which we list them is convention, not fact.

An admissible measurement protocol π is any finite sequence of physical interactions the observer can perform. Let $X_H(\pi)$ be the outcome distribution given history H. Define

$$d(H_1, H_2) = \sup_{\pi \text{ admissible}} D_{\mathrm{TV}}(X_{H_1}(\pi), X_{H_2}(\pi)), \tag{1}$$

with D_{TV} total variation. Operational equivalence $H_1 \sim H_2$ iff $d(H_1, H_2) = 0$. The physical state space:

$$S_{\mathcal{O}} = \mathcal{H}/\sim . \tag{2}$$

The equivalence relation is fixed. The quotient $\sim$ is determined once and for all by the observer's physical constitution (O1, O2, O6) and the resulting class of admissible protocols. It is not a dynamical variable. A system whose distinguishability rules change under interaction — where two histories that the original equivalence identifies become distinguishable later, or vice versa — is not a single persistent observer but a sequence of distinct ones. O2 (fixed memory register) and O6 (continuous proper-time evolution) together guarantee persistent observer identity over the worldline. Multi-observer constructions in Part III apply to admissible families each of whose members carries a fixed equivalence relation; adaptive systems whose internal coarse-graining evolves are absorbed into the framework as additional degrees of freedom of the substrate, not as observers in the OFP sense.

Convex closure (operational assumption). If preparation procedures P_1, P_2 are admissible, a randomised choice between them with probabilities $\lambda, 1-\lambda$ is also admissible. This induces a convex structure on $S_{\mathcal{O}}$: states $s_1, s_2 \in S_{\mathcal{O}}$ admit convex combinations $\lambda s_1 + (1-\lambda)s_2 \in S_{\mathcal{O}}$. We treat this as an operational assumption rather than a derived consequence; without it, the later identification of probability measures requires additional input.

Proposition 2.1 (Symmetric composition)**.** *For causally disconnected sectors A, B: $\mathcal{H}_A \circ \mathcal{H}_B \sim \mathcal{H}_B \circ \mathcal{H}_A$. Restricted to such sectors, $(S_{\mathcal{O}}, \oplus)$ is a commutative topological monoid, locally compact (O3) and Hausdorff (quotient metric).*

Lemma 2.2 (Cancellativity)**.** *If $[H_1] \oplus [H_3] = [H_2] \oplus [H_3]$ for any causally independent H_3, then $[H_1] = [H_2]$.*

Proof. Let π be any admissible protocol. By O5, the observer's apparatus cannot contact H_3 during π, so the protocol's statistics on $H_1 \oplus H_3$ are determined entirely by H_1. Hypothesis gives agreement, so π does not distinguish H_1 from H_2. Since π was arbitrary, $H_1 \sim H_2$. □

Lemma 2.3 (Translation invariance of d)**.** *$d(s \oplus t, s' \oplus t) = d(s, s')$ for causally independent s, s', t.*

Corollary 2.4 (Grothendieck group)**.** *The Grothendieck group $G(S_{\mathcal{O}})$ is a locally compact Hausdorff Abelian group, into which $S_{\mathcal{O}}$ embeds as the positive cone.*

Positive-cone embedding. The Grothendieck group $G(S_\mathcal{O})$ consists of formal differences $[s]-[t]$ of physical states. Although $[s]-[t]$ need not itself correspond to a preparable state, the embedding $\iota : S_\mathcal{O} \to G(S_\mathcal{O})$, $\iota([s]) = [s]-[\emptyset]$, is injective: if $\iota([s]) = \iota([s'])$ then by cancellativity (Lemma 2.2) $[s] = [s']$. The image $\iota(S_\mathcal{O})$ is the positive cone; it generates G by construction. Elements outside the positive cone are bookkeeping artefacts of the group completion, retained only to access the Pontryagin structure theorem; no physical claim is made about them.

Completeness. A0 requires completion: a Cauchy sequence under d has the property that no admissible protocol distinguishes sufficiently late elements. Refusing to identify the limit with a state would assert an operationally invisible distinction, which A0 forbids.

3 Real Vector Space Structure

Physical picture. Pull together what we have: physical states form a set you can scale continuously, combine with independent things, and cancel reversibly. That is what a vector space is.

Path-connectedness. Every state is reached from $[\emptyset]$ by a finite interaction sequence (O4). Each interaction is continuous (O6). Hence $G(S_\mathcal{O})$ is path-connected.

Torsion elimination. Suppose $n\theta = 0$ with $\theta \neq 0$. By Lemma 2.3, $d(k\theta, (k-1)\theta) = d(\theta, 0) > 0$ for each k, so by triangle inequality $d(n\theta, 0) > 0$, contradicting $n\theta = 0$. Operational continuity (O6) excludes discrete finite-order cycles in connected sectors.

Topological assumptions for Pontryagin. The structure theorem applies to a Hausdorff, second-countable, locally compact, connected Abelian group. Hausdorff and locally compact are inherited from the operational metric and O3. Second countability is implicit in the finite-protocol-history specification (O4 plus the countability of admissible protocols). Path-connectedness is established above. We state these as operational assumptions consistent with the framework rather than derive them from O1–O6 alone.

By the Pontryagin structure theorem, a complete, torsion-free, locally compact, path-connected, second-countable Abelian group is a finite-dimensional real vector space:

$$S_\mathcal{O} \hookrightarrow G(S_\mathcal{O}) \cong \mathbb{R}^n, \quad 2 \le n \le k. \tag{3}$$

4 The Euclidean Norm and Inner Product

Physical picture. Splitting a system into independent parts and recombining them is passive redescription. Changing the orthonormal measurement basis is exactly such a split. The distinguishability measure must be invariant under any orthogonal change of basis. The only norm with that invariance is Euclidean.

Lemma 4.1 ($O(n)$-invariance of the distinguishability norm)**.** *The norm $\|x\| = d(x, 0)$ on $\mathbb{R}^n$ is $O(n)$-invariant.*

Proof. Let $R \in O(n)$ be any orthonormal rotation. R maps one orthonormal basis $\{e_i\}$ of $\mathbb{R}^n$ to another $\{Re_i\}$. Both are complete orthonormal descriptions of the same physical state space, so $x \mapsto Rx$ is a change of basis label, not a change of physical state. An admissible measurement protocol π is a finite sequence of local physical interactions (O4); the interaction Hamiltonian couples to physical states, not to the label of which orthonormal decomposition is written down. The observer's memory register records a classical outcome index — a discrete label written by the physical interaction. Relabelling which index corresponds to which basis element is a passive permutation of symbols leaving the physical interaction, and the register's physical state after recording, unchanged. Hence for every admissible π there exists an admissible π' (the same sequence with the orthonormal relabelling removed from the recording step) such that

$$D_{\mathrm{TV}}(X_{Rx}(\pi), X_0(\pi)) = D_{\mathrm{TV}}(X_x(\pi'), X_0(\pi')).$$

The map $\pi \mapsto \pi'$ is a bijection on admissible protocols, so the suprema agree: $d(Rx, 0) = d(x, 0)$. $\square$

By Lemma 4.1 the norm is $O(n)$-invariant; by Lemma 1.3 it is therefore Euclidean, and $\|x \oplus y\|^2 = \|x\|^2 + \|y\|^2$. By the parallelogram law and the Jordan–von Neumann theorem [9]:

$$\langle x, y\rangle = \tfrac{1}{4}(\|x + y\|^2 - \|x - y\|^2). \tag{4}$$

5 The Phase Group is U(1)

Physical picture. An isolated system carries internal motion that does not change probabilities (Step 4: no interaction, no information). Free dynamics decompose the state space into rotating two-dimensional planes. If two planes are coupled by any admissible interaction, their phases must rotate together: otherwise the interaction itself would detect the relative phase, making independent rotation physically non-inert. If two planes are completely decoupled, their relative phase is gauge and already eliminated by the operational quotient. Either way, exactly one global U(1) phase freedom remains.

Free evolution preserves the Euclidean norm, hence is a one-parameter subgroup of $O(n)$ of the form $\exp(tA)$ with A skew-symmetric. By the real canonical form, A decomposes $\mathbb{R}^n$ into two-dimensional invariant planes $\Pi_1, \ldots, \Pi_m$.

Lemma 5.1 (Phase group from pairwise structure)**.** *The unique physically inert norm-preserving freedom is the global rotation $\psi \mapsto e^{i\theta}\psi$.*

Proof. Define a graph G on $\{1, \ldots, m\}$ with edge (j, k) iff some admissible observable M has nonzero cross-block M_{jk} between Π_j and Π_k.

Coupled pairs. For $\psi = \psi_j + \psi_k$ in distinct planes under independent rotation (θ_j, θ_k) with $\Delta\theta = \theta_j - \theta_k \neq 0$:

$$\langle\psi|M|\psi\rangle \to \langle\psi_j|M|\psi_j\rangle + \langle\psi_k|M|\psi_k\rangle + 2\,\mathrm{Re}\big(e^{i\Delta\theta}\langle\psi_j|M|\psi_k\rangle\big),$$

which depends on $\Delta\theta$. By A0 the transformation is not inert. Hence $\theta_j = \theta_k$ for every coupled pair.

Uncoupled pairs. All admissible expectations on $a\psi_j + b\psi_k$ depend only on $|a|, |b|$. Relative phase changes no admissible expectation; it is gauge and already quotiented out in $S_{\mathcal{O}}$.

Combining. Within each connected component of G Branch I forces synchrony; across components Branch II makes the relative phase gauge. The total inert freedom is one real parameter regardless of G's structure. Phase group $= U(1)$. $\square$

Numerical verification. The dichotomy is verified directly in Appendix A on a 4-dimensional toy space. Branch I: $(\theta_1, \theta_2) = (0, \pi/4)$ changes the off-diagonal observable's expectation from 1.000 to 0.707 — operationally distinguishable. Branch II: eight independent choices of (θ_1, θ_2) all yield identical expectation 0.250.

6 The Scalar Field is $\mathbb{C}$, and Hilbert Space

Physical picture. The scalar field must be a finite-dimensional associative division algebra over $\mathbb{R}$, commutative (from symmetric composition of disconnected sectors), and contain a $U(1)$ circle. The Frobenius theorem leaves only one option.

The scalar field K must be a finite-dimensional associative division algebra over $\mathbb{R}$. Division-algebra requirement: if some nonzero $c \in K$ failed to be invertible, then $cv = 0$ for some nonzero v — a pure redescription scaling a nonzero state to zero, contradicting A0.

Commutativity follows from Lemma 5.1: scalar multiplication by unit-modulus elements must commute with the diagonal $U(1)$ action, factoring through the commutative subalgebra $\mathbb{C} \subset \mathrm{End}_{\mathbb{R}}(\Pi_j)$.

By the Frobenius theorem [10], the only options are $\mathbb{R}, \mathbb{C}, \mathbb{H}$. $\mathbb{R}$ has discrete unit group; $\mathbb{H}$ is non-commutative. Therefore $K = \mathbb{C}$.

The $U(1)$ generator J on each plane satisfies $J^2 = -I$. Defining

$$\langle x, y\rangle_{\mathbb{C}} = \langle x, y\rangle_{\mathbb{R}} + i\langle Jx, y\rangle_{\mathbb{R}} \tag{5}$$

gives a sesquilinear conjugate-symmetric positive-definite form, so $\mathcal{H}_{\mathcal{O}} \cong \mathbb{C}^m$ is a finite-dimensional complex Hilbert space.

7 Tensor Product Composition

Physical picture. For two causally independent systems, the only operationally accessible information about the joint state is what local measurements during the period of causal independence reveal. Any degree of freedom invisible to all local measurements during that period is quotiented out by A0.

Operational framing. The argument below treats local tomography as a consequence of the restricted protocol structure during causal independence, not as an independent axiom. The premise is operational: during the disconnection epoch, by O5, no joint interaction unitary is realisable; the available admissible protocols decompose into local operations on A, local operations on B, and classical communication. Joint states are characterised by such protocols. Whether this premise can be weakened — whether richer joint structures might survive A0 under a broader notion of admissible protocol — is part of the completeness question (Sec. 31, Open Problem 1).

Lemma 7.1 (Local tomography)**.** *If two states σ, σ' of a composite system whose sectors are causally disconnected during preparation agree on every statistic obtainable by local operations and classical communication, then $\sigma = \sigma'$.*

Proof. Suppose $\sigma \neq \sigma'$. By Step 5 some admissible protocol distinguishes them. During causal independence (O5), the only available operations are local plus classical communication; by hypothesis these agree, so no admissible protocol distinguishes them — contradiction. □

Theorem 7.2 (Tensor composition)**.** *For causally independent systems A, B: $\mathcal{H}_{AB} \cong \mathcal{H}_A \otimes \mathcal{H}_B$.*

Proof. Joint preparation is bilinear by linearity of admissible-protocol statistics in each factor (Step 5). By Lemma 7.1, overlaps with product states determine all states, so the universal property of the tensor product is satisfied. No vector unreachable from products has admissible preparation; no product state is excluded by causal independence. □

8 The Born Rule and Unitary Dynamics

Physical picture. The probability of a measurement outcome should depend only on which outcome is being asked about, not on what other outcomes happen to be available. That context-independence, combined with normalisation, defines a Gleason frame function. Free evolution preserves the inner product, so by Wigner's theorem is unitary or anti-unitary; continuous proper time excludes anti-unitary.

8.1 Born rule

The observer is always present (Step 1) with at least two distinguishable internal states (O3), so any real measurement happens on the joint system $\mathcal{H}_{\text{joint}} = \mathcal{H}_{\mathcal{O}} \otimes \mathcal{H}_S$ of dimension $\geq 4 \geq 3$. This closes the qubit gap structurally: prior to interaction, observer and system are causally independent (Step 8) so the joint space is exactly $\mathcal{H}_{\mathcal{O}} \otimes \mathcal{H}_S$ (Theorem 7.2).

Move (i): frame function. Two orthonormal bases of $\mathcal{H}_{\text{joint}}$ sharing a ray e_1 differ by an orthogonal transformation that fixes P_{e_1}. By A0 this is physically inert at the e_1 outcome. Hence $P(e_1)$ depends only on P_{e_1}, not on the basis context within $e_1^{\perp}$. This is precisely Gleason's definition of a frame function.

Move (ii): Gleason. Since $\dim \mathcal{H}_{\text{joint}} \geq 3$*, Gleason's theorem [12] gives*

$$P(E) = \operatorname{Tr}(\rho_{\mathcal{O}S} P_E) \tag{6}$$

for some joint density operator $\rho_{\mathcal{O}S}$. The system Born rule follows by partial trace:

$$P(E_S) = \operatorname{Tr}_S[\rho_S P_{E_S}], \quad \rho_S = \operatorname{Tr}_{\mathcal{O}} \rho_{\mathcal{O}S}. \tag{7}$$

Status of the Born rule. The Born rule is not derived from A0 alone. A0 plus the operational state space construction establishes the frame-function property of any context-independent probability assignment on the joint Hilbert structure; Gleason's theorem (imported) then forces the trace form. The qubit-gap closure — the system Born rule via partial trace from a $\dim \geq 4$ *joint space — is the OFP-specific contribution.*

8.2 Unitary dynamics

The unitary group $U(\mathcal{H})$ is the identity component of the group of all isometries of $\mathcal{H}$; anti-unitary operators form the other component. By Wigner's theorem [13], free evolution is unitary or anti-unitary. A continuous one-parameter group through the identity (O6) must remain in the identity component. By Stone's theorem [14]:

$$U(\tau) = e^{-iH\tau/\hbar}, \quad H \textit{ self-adjoint.} \tag{8}$$

H is the observer's clock generator (used in Parts II and IV).

9 Classical Action Equivalence

Physical picture. The Madelung transformation is a pure A0-invariant rewriting of the Schrödinger equation into classical variables. The resulting equations are exactly equivalent to quantum dynamics when the classical action is allowed to be multi-valued.

The substitution $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ maps the Schrödinger equation exactly to the continuity equation for ρ and the Hamilton–Jacobi equation for S with quantum potential

$$Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}.$$

Bohm [16] (and recent work of Lohmiller–Slotine [17] in the relativistic case) establishes the exact converse: given the multi-valued classical extremal action and the density along each branch, $\psi = \sum_j \sqrt{\rho_j} e^{i\phi_j/\hbar}$ satisfies the Schrödinger equation exactly. Branch points of the multi-valued action — multiply-connected manifolds (double slit), spatial constraints, singularities — are the classical mechanism producing quantum interference. This is exact, not semiclassical.

The classical-action route supplies amplitudes; the observer-first route supplies the operational meaning of probabilities. The two are exactly complementary at the level of dynamical representation; this is representational equivalence, not ontological reduction.

10 Electromagnetism from Local Phase Freedom

Physical picture. The global $U(1)$ phase rotation is inert. But an observer is local and can only enforce a phase convention along their own worldline. No physical mechanism imposes a single global phase across spacelike-separated regions; doing so would require instantaneous influence (O5). The inert transformation is therefore arbitrary smooth local phases $\theta(x,\tau)$, not merely the global constant. Requiring the action to remain stationary forces a compensating gauge field, which turns out to be the Maxwell field.

Under $\psi(x,\tau) \mapsto e^{i\theta(x,\tau)}\psi$, the ordinary derivative transforms inhomogeneously. Introduce A_μ with $A_\mu \to A_\mu + (1/e)\partial_\mu\theta$. The covariant derivative $D_\mu = \partial_\mu - ieA_\mu$ then transforms covariantly.

Theorem 10.1 (Gauge structure)**.** *The minimal structure restoring stationarity under local $U(1)$ is $D_\mu = \partial_\mu - ieA_\mu$ with kinetic term $S_{\text{EM}} = -\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}d^4x$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, yielding Maxwell's equations. The gauge boson is massless: a Proca mass term breaks $A_\mu \to A_\mu + (1/e)\partial_\mu\theta$ and so violates A0.*

Two named principles. Minimality (Occam-type, not from A0) and Lorentz invariance (locally implicit in Steps 2 and 7, globally supplied by Part III, Sec. 16). Theorem 10.1 should accordingly be read as a consequence of A0 + locality + PLA + minimality + Lorentz invariance.

The numerical value of e remains a free parameter set by experiment. Identifying this gauge structure with Maxwell electrodynamics is empirical; identifying the non-Abelian extensions with the Standard Model gauge groups is empirical and outside the scope of the constraint-based framework.

11 Arrow of Time, Second Law, and Wave Collapse

The strict causal ordering of the finite memory register (O2) constitutes the observer's internal arrow of time. When the register is full, new records require erasure. By Landauer's principle [20], erasure of one bit dissipates at least $k_BT\ln 2$:

$$\frac{dS_{\text{env}}}{d\tau} \geq k_B \ln 2 \cdot r > 0 \tag{9}$$

where r is the bit-erasure rate. The Second Law is a direct consequence of finite record-keeping by a local observer, not an independent thermodynamic postulate.

In the observer-first framework, wave collapse is the same event as an irreversible write to the finite memory register. The write is irreversible (O2 plus Landauer); it selects a specific branch from the multi-valued action; it produces entropy in the environment. Arrow of time, Born rule, and wave collapse are three aspects of the same constraint.

The framework establishes the structure *of measurement. The* interpretation — *whether the record-write creates the outcome or reveals it — is empirical and outside scope.*

Part II

Observer-Realizable Channels: Operational Bounds

12 The Operational Scales

Physical picture. The observer's memory records events by transitioning between orthogonal states. The Margolus–Levitin theorem sets the shortest time for such a transition given the available energy. This is the observer's fundamental temporal resolution — not an engineering limitation but a consequence of finite energy and the quantum nature of the memory register. The Shannon–Nyquist theorem then limits the frequencies the observer's control signals can faithfully represent.

Proposition 12.1 (Temporal resolution)**.** *The minimum proper time for a distinguishable memory update is*

$$\tau_{\mathcal{O}} = \frac{\pi\hbar}{2E_{\mathrm{op}}}. \tag{10}$$

Proof. By the Margolus–Levitin theorem [18], a quantum system with average energy E_{op} above its ground state requires at least $\pi\hbar/(2E_{\mathrm{op}})$ to evolve to an orthogonal state. Memory update by O2 requires distinct (orthogonal) states; the bound applies. □

For a transmon qubit with level spacing $E_{\mathrm{op}}/h = 1$ GHz, $\tau_{\mathcal{O}} \approx 250$ ps. At $E_{\mathrm{op}}/h = 10$ MHz, $\tau_{\mathcal{O}} = 25$ ns.

Proposition 12.2 (Control bandwidth)**.** *A control signal generated by the observer's substrate cannot be updated faster than $\tau_{\mathcal{O}}^{-1}$. By Shannon–Nyquist:*

$$\Lambda_{\mathcal{O}} = \frac{1}{2\tau_{\mathcal{O}}} = \frac{E_{\mathrm{op}}}{\pi\hbar}. \tag{11}$$

Proposition 12.3 (Timing-jitter floor)**.** *To operationally distinguish a pulse at t from a pulse at $t + \delta t$, the observer must record them as distinct events in orthogonal memory states. Hence $\sigma_t \geq \tau_{\mathcal{O}}$ regardless of engineering quality.*

These three scales are forced from O1 + O2 alone (plus Margolus–Levitin and Nyquist as imports). They have no free parameters.

13 Observer-Realizable Channels

Physical picture. Any quantum channel a real observer can implement is constrained by their memory capacity, energy, and bandwidth. These constraints define the class of Observer-Realizable Channels (ORCs).

Definition 13.1 (ORC)**.** *A CPTP map $\mathcal{E} : \mathcal{B}(\mathcal{H}_S) \to \mathcal{B}(\mathcal{H}_S)$ is an ORC of resources (M, E_{op}) if it admits a Stinespring dilation*

$$\mathcal{E}(\rho) = \mathrm{Tr}_{\mathcal{O}}\big[U(\rho \otimes |0\rangle\langle 0|_{\mathcal{O}})U^{\dagger}\big]$$

with: ancilla dimension $\leq M$, interaction Hamiltonian $\|H_{\mathrm{int}}(t)\| \leq E_{\mathrm{op}}$, interaction time $T \geq \tau_{\mathcal{O}}$, and $\int_{|\omega|>2\pi\Lambda_{\mathcal{O}}} \|\tilde{H}_{\mathrm{int}}(\omega)\|^2 d\omega \leq \varepsilon$, with ε exponentially small for optimally shaped pulses.

Theorem 13.2 (Bounds on ORCs)**.** *For any ORC $\mathcal{E}$ at resources (M, E_{op}):*

1. *Kraus rank* $\leq M$.
2. *Minimum gate time for target unitary* V *with spectral diameter* θ*:* $T \geq \max(\tau_{\mathcal{O}}, \hbar\theta/(2E_{\mathrm{op}}))$.
3. *Bandwidth-limited infidelity:* $1 - F \gtrsim \exp(-c\Lambda_{\mathcal{O}} T)$ *with* $c \approx 2\pi$ *for optimally band-limited control.*

The Kraus rank bound follows from the ancilla dimension; the gate-time bound from Mandelstam–Tamm applied to $\|H_{\mathrm{int}}\| \leq E_{\mathrm{op}}$*; the infidelity bound from the prolate spheroidal decomposition of optimally band-limited pulses.*

Proposition 13.3 (Landauer dissipation)**.** *With erasure rate at most* $M/\tau_{\mathcal{O}}$*, each erased bit dissipating at least* $k_B T_{\mathrm{env}} \ln 2$*:*

$$P_{\mathcal{O}} \geq k_B T_{\mathrm{env}} \ln 2 \cdot \frac{M}{\tau_{\mathcal{O}}}. \tag{12}$$

14 Dynamical Decoupling: A Falsifiable Prediction

Physical picture. When an observer protects a qubit from $1/f$ *noise using a CPMG pulse train, they apply* n π*-pulses spaced* $\Delta t = T/n$ *apart. Timing jitter with variance* σ_t^2 *smears the filter function. The jitter floor* $\sigma_t \geq \tau_{\mathcal{O}}$ *means that beyond some optimal pulse number* n_{opt}*, adding more pulses worsens coherence rather than improving it. This non-monotonic behaviour, with specific dependence on drive energy, is experimentally testable and does not appear in standard DD theory.*

For a qubit subject to $S_\beta(\omega) = A/|\omega|$ *noise with high-frequency cutoff* $\tau_c = 1$ *ns and total time* $T = 1\,\mu s$*, the ensemble-averaged filter function with Gaussian jitter* $\delta t_k \sim \mathcal{N}(0, \sigma_t^2)$*,* $\sigma_t \geq \tau_{\mathcal{O}}$*, gives a dephasing exponent*

$$\chi(n) \simeq \frac{AT^2}{2}\frac{\ln n}{n^2} + \frac{4An\sigma_t^2}{\pi}\ln\frac{T}{\tau_c}. \tag{13}$$

The first term decreases with n *(ideal CPMG suppression); the second increases with* n *(jitter accumulation). Minimisation yields the self-consistent equation*

$$n_{\mathrm{opt}}^3 \simeq \frac{\pi T^2}{2\sigma_t^2 \ln(T/\tau_c)} \ln n_{\mathrm{opt}}. \tag{14}$$

Table 1: Optimal CPMG pulse number from Eq. (14) with $\sigma_t = \tau_{\mathcal{O}} = \pi\hbar/(2E_{\mathrm{op}})$, $\ln(T/\tau_c) \approx 6.908$.

E_{op}/h (GHz)	n_{opt}
0.1	53
0.5	166
1	279
5	839
10	1366
50	4176
100	6760

A power-law fit gives $n_{\mathrm{opt}} \propto E_{\mathrm{op}}^{0.70}$*. The leading-order naive scaling* $E_{\mathrm{op}}^{2/3}$ *receives a calculable logarithmic correction from the* $\ln n_{\mathrm{opt}}$ *factor in Eq. (14), lifting the effective exponent above* $2/3$*. This is verified directly in Appendix A (Toy 1).*

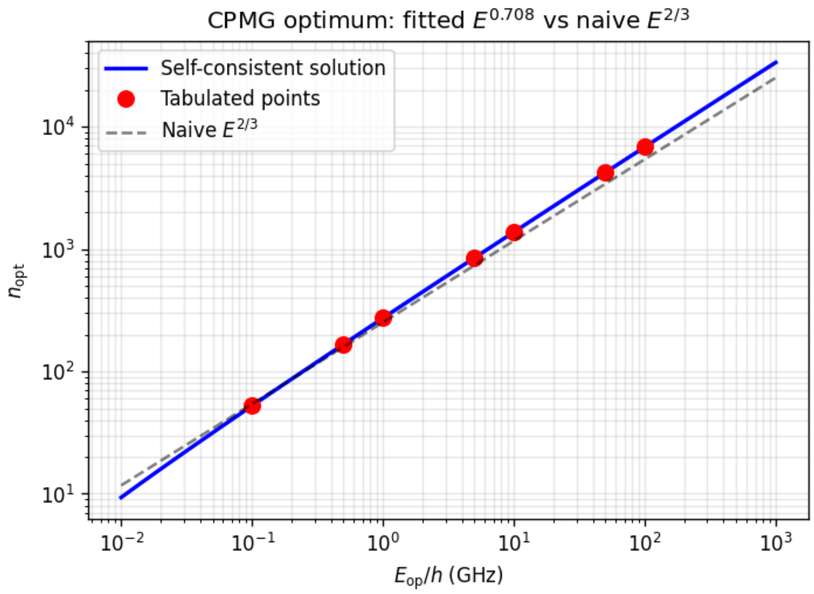

Figure 1: Self-consistent solution of Eq. (14) for $n_{\rm opt}$ vs $E_{\rm op}/h$ (solid blue), with table values from Table 1 marked (red circles). Naive $E_{\rm op}^{2/3}$ scaling shown for comparison (dashed); the actual exponent 0.70 is calibration-free.

Coherence is predicted to decrease *for $n > n_{\rm opt}$, with a specific energy-dependent location of the optimum. This non-monotonic behaviour, with no free parameters, is the falsifiable signature of the finite-observer framework, testable with current transmon technology.*

Null hypothesis and discrimination

Standard DD prediction. In the absence of a fundamental jitter floor, CPMG coherence is monotonically improved by increasing n until it saturates at hardware-limited timing jitter (typically $\sigma_t^{\rm hw} \sim 10$–$100$ ps for room-temperature electronics, ~ 1 ps for cryogenic AWG). Any optimum is set by hardware, with no specific scaling against drive energy $E_{\rm op}$.

OFP prediction. A fundamental optimum exists at the $E_{\rm op}$-dependent value of Table 1, with scaling $n_{\rm opt} \propto E_{\rm op}^{0.70}$, independent of hardware quality. Sweeping $E_{\rm op}$ across e.g. 100 MHz to 10 GHz drive amplitudes and tracking $n_{\rm opt}$ at fixed T_2^* provides the discrimination: standard theory gives a flat or hardware-dominated optimum; OFP gives the indicated power-law scaling.

Existing transmon parameters. Typical superconducting transmon qubits have $T_2^* \sim 50$–$200\,\mu$s and accept drives in the 500 MHz–5 GHz range; the table values lie within experimentally accessible n. Distinguishing the two predictions requires resolution of the optimum location to $\sim 30\%$ across one decade of $E_{\rm op}$, which is feasible with existing protocols.

Distinct from existing non-monotonic observations. Reports of non-monotonic CPMG behaviour in the literature attribute the optimum to specific control imperfections (pulse-shape errors, microwave phase noise) without an energy-scaling law. The OFP prediction is that the optimum persists — with the indicated $E_{\rm op}^{0.70}$ scaling — as those engineering imperfections are reduced.

Part III

Multi-Observer Spacetime and General Relativity

15 A0$^+$ and the Causal Order

Physical picture. When many observers interact, the invariant structure across all of them defines spacetime. Closed timelike curves are excluded because they would require an observer's memory register to contain entries both before and after the same event simultaneously.

Definition 15.1 (Admissible family)**.** *An admissible family $\mathcal{F}$ is a set of observers, each satisfying O1–O6, such that any two members are connected by a finite chain of admissible physical interactions.*

Definition 15.2 (A0$^+$)**.** *A transformation is physically inert relative to $\mathcal{F}$ iff it changes no outcome of any physical process for any member of $\mathcal{F}$. Physical reality consists solely of what is invariant under all such transformations. $|\mathcal{F}| = 1$ recovers A0.*

For events e_1, e_2 in the family, define $e_1 \preceq e_2$ if a chain of signals at speed $\leq c$ (O5) connects them.

Proposition 15.3 (No CTCs)**.** *The relation $\preceq$ is a strict partial order: a closed chain would force an observer's memory (O2) to contain an entry both before and after itself, contradicting strict causal ordering.*

16 Lorentzian Signature

Physical picture. Given a smooth pseudo-Riemannian substrate, the causal structure forces a partition of tangent vectors into timelike, null, and spacelike. Only Lorentzian signature can accommodate exactly one timelike direction per observer while maintaining a sharp causal boundary.

Proposition 16.1 (Lorentzian signature is forced)**.** *Any smooth pseudo-Riemannian substrate consistent with (i) the causal partial order of A0$^+$, (ii) finite signal speed (O5), and (iii) exactly one continuous timelike direction per observer must have Lorentzian signature.*

The "exactly one" clause derives from the unity of the observer. By O2, an observer has a single linearly-ordered memory register; by O6, a single continuous proper-time parameter; by O1, a single physical system. Hence the worldline direction at any point is unique.

Proof. Riemannian (+,+,+,+): all vectors spacelike, no light-cone. Excluded.

Degenerate: no sharp causal boundary. Excluded.

Split (2,2): two-dimensional timelike plane at each point, giving a two-parameter family of timelike directions, contradicting (iii). Excluded. This is verified directly in Appendix A (Toy 3).

Lorentzian $(-,+,+,+)$: exactly one timelike direction; admissible. The dual $(+,-,-,-)$ is related by overall sign reversal, a passive redescription. □

Remark 16.2 (Lorentz invariance import for Theorem 10.1)*.* Theorem 10.1 cited Lorentz invariance as an additional input. Proposition 16.1, read in the joint framework, supplies the global Lorentzian signature from the causal partial order, finite signal speed, and the single-timelike-direction condition — given the assumed manifold. The kinetic-term uniqueness of the EM theorem rests on Lorentz invariance that this proposition supplies.

17 Conformal Class and the Full Metric

Physical picture. The causal order alone fixes the metric up to a local conformal factor. Each observer's own clock — their proper time — supplies the missing scale.

Strong causality follows from the no-CTC property: a failure of strong causality would produce a closed causal curve in the limit. By Malament's theorem [25], any causal-order-preserving bijection between strongly causal Lorentzian manifolds is a smooth conformal isometry. The causal order therefore fixes the metric up to $g_{\mu\nu} \mapsto \Omega^2(x) g_{\mu\nu}$*.*

The proper-time normalisation $g_{\mu\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu = -c^2$ *along observer worldlines (O6) pins* Ω *at each worldline point; smoothness extends* Ω *everywhere. Conformal class plus proper-time scaling determines* $g_{\mu\nu}$ *uniquely.*

18 The Equivalence Principle

Physical picture. At any point you can choose coordinates that make the metric flat to first order. Changing to those coordinates is a passive relabelling. By A0$^+$ *it is inert. Therefore local physics cannot detect the difference between curved and flat spacetime to first order in derivatives.*

Proposition 18.1 (Equivalence principle)**.** *For any observer worldline point p, Riemann normal coordinates achieve* $g_{\mu\nu}(p) = \eta_{\mu\nu}$ *and* $\partial_\rho g_{\mu\nu}(p) = 0$*. The coordinate transformation is a passive redescription, hence inert by A0*$^+$*. No admissible protocol confined to a small neighbourhood detects curvature to first order.*

Higher-order curvature effects (the Riemann tensor) are not zeroed by Riemann normal coordinates and constitute the genuinely physical content of curvature. We work exclusively at the passive level; the hole-argument debate about active diffeomorphisms is taken with the standard resolution.

19 The Closure Condition

Physical picture. Each observer in the family is a physical system: matter with finite energy. Other observers see them as part of the world. So the substrate includes the family. This is where the source term for the field equations comes from — and where the clock generator of Part IV is forced into the local algebra.

Proposition 19.1 (Closure)**.** *Each observer* $\mathcal{O} \in \mathcal{F}$ *is a physical system with finite energy and stress-energy distribution (O1).* $\mathcal{O}$*'s stress-energy contributes to the substrate that other family members perceive. The total* $T_{\mu\nu}$ *includes all matter, both observers and any non-observer matter the family interacts with.*

20 The Bekenstein Bound, Horizon Entropy, and Einstein Equations

Physical picture. The Bekenstein bound limits information in any region. At gravitational collapse the saturating entropy is the area law. Applying the Clausius relation to local Rindler horizons with the Unruh temperature, together with the Raychaudhuri equation, yields Einstein's equations via Jacobson's thermodynamic construction. In a calculable 1+1-dimensional Rindler model the microscopic Landauer cost of memory erasure matches the macroscopic heat flow term by term.

20.1 Imports

- *Bekenstein bound [26]:* $S \leq 2\pi RE/(\hbar c)$.
- *Collapse limit:* $E_{\max} = Rc^4/(2G)$. *Saturation gives* $S = A/(4\ell_P^2)$.
- *Unruh temperature for local Rindler horizons:* $T_U = \hbar a/(2\pi c k_B)$ *[27].*
- *Bisognano–Wichmann [29]: modular flow on a Rindler wedge equals the boost.*
- *Raychaudhuri equation for null congruences.*

20.2 The Landauer–Clausius bridge in 1+1D

Take a uniformly accelerated observer with proper acceleration a *in 1+1-dimensional Minkowski. The Rindler horizon is the null ray* $x = t$. *By Bisognano–Wichmann, the modular Hamiltonian of the wedge is the boost generator*

$$K = \frac{2\pi}{\hbar} \int dx\, x\, T_{tt}(x). \tag{15}$$

A matter perturbation $T_{\mu\nu}$ *crossing the horizon changes*

$$\delta K = \frac{2\pi}{\hbar} \int T_{\mu\nu} k^\mu k^\nu d\lambda. \tag{16}$$

This is new information entering the observer's causal patch. By the Bekenstein bound, the entropy of this perturbation satisfies $\delta S = \delta K/T_U$. *The observer's memory must accommodate this information. The Landauer cost of registering and subsequently erasing these bits at temperature* T_U *is*

$$\delta Q_{\text{Landauer}} = T_U \cdot \delta S = T_U \cdot \frac{\delta K}{T_U} = \delta K = \frac{2\pi}{\hbar} \int T_{\mu\nu} k^\mu k^\nu d\lambda. \tag{17}$$

In 1+1D the transverse integral is trivial, so this equals the full heat flow $\delta Q = \int T_{\mu\nu} k^\mu k^\nu d\lambda$. *The Clausius relation is recovered term by term from Landauer erasure costs.*

This argument requires $\delta S \leq M \ln 2$ *— the perturbation must fit within memory. This is precisely the semiclassical regime in which the Einstein equations are valid. The condition and the domain of validity coincide exactly. This is verified numerically in Appendix A (Toy 4).*

20.3 Einstein's equations

For a local Rindler horizon at any point p *in any null direction* k^μ*:*

- *Heat flow:* $\delta Q = \int T_{\mu\nu} k^\mu k^\nu d\lambda dA$.
- *Area change governed by Raychaudhuri:* $d\theta/d\lambda = -\frac{1}{2}\theta^2 - \sigma^2 - R_{\mu\nu} k^\mu k^\nu$ *(shear and twist suppressed at the horizon).*
- *Clausius* $\delta Q = T_U dS$ *with* $dS = dA/(4\ell_P^2)$.

Theorem 20.1 (Einstein's equations, conditional on imports)**.** *Under* $A0^+$, *O1–O6, the smooth manifold structure, the equivalence principle, the Bekenstein bound, the Unruh temperature, and the Raychaudhuri equation, the substrate metric satisfies*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{18}$$

with Λ *an undetermined integration constant.*

The proof is Jacobson's 1995 thermodynamic construction [30] with the Clausius relation supplied operationally by the Landauer–Clausius bridge above. Newton's constant G *enters as an empirical input through the area law.*

Part IV

Type II Algebraic Quantum Field Theory

21 Finite-Observer Algebras and the Clock in the Algebra

Physical picture. An observer with finite energy can only access a finite number of field modes in their laboratory. The resulting algebra of observables is a finite-dimensional matrix algebra. As their energy resources grow, they access progressively more of the field. The observer's clock must be part of the physical algebra: if it were not, physically distinct timing records would be assigned identical statistics, violating A0.

For a free scalar field on the substrate of Part III, restrict to modes with $\omega \le 2\pi\Lambda_{\mathcal{O}}$ *and support in region* $\mathcal{O}$. *The generated von Neumann algebra is* $\mathfrak{A}_{(\Lambda_{\mathcal{O}},\mathcal{O})}$, *a finite-dimensional type I factor. As the energy bound is raised, the weak closure of the union of these algebras recovers the standard Rindler algebra.*

Proposition 21.1 (Clock generator is in the algebra)**.** *The observer's clock Hamiltonian* $H_{\mathcal{O}}$ *with* $\|H_{\mathcal{O}}\| \le E_{\mathrm{op}}$ *(Stone, O1, O6) is an element of the physical algebra.*

Proof. By O2, two situations differing only in proper-time records have operationally distinguishable statistics. If $H_{\mathcal{O}}$ were not in the physical algebra, no operator in the algebra would distinguish them, contradicting A0. Hence $H_{\mathcal{O}} \in \mathfrak{A}_{\mathrm{phys}}$. □

22 The Physical Algebra is the Fixed-Point Algebra

Physical picture. Shifting the observer's proper-time origin is a passive redescription — it amounts to choosing when to start the clock. By A0 this is physically inert. Physical observables must therefore be invariant under clock shifts: the physical algebra is the set of operators that commute with $H_{\mathcal{O}}$.

Lemma 22.1 (Clock-shift inertness)**.** *For any observer* $\mathcal{O} \in \mathcal{F}$ *with clock generator* $H_{\mathcal{O}}$, *the one-parameter automorphism group* $\sigma_\alpha^{\mathrm{clock}}(a) = e^{i\alpha H_{\mathcal{O}}} a e^{-i\alpha H_{\mathcal{O}}}$ *is physically inert under A0.*

Proof. The shift $\tau \mapsto \tau + \alpha$ is a passive coordinate redescription of $\mathcal{O}$'s proper-time axis, analogous to the passive coordinate inertness of Section 18. The unitary $e^{i\alpha H_{\mathcal{O}}}$ implements this redescription; $\sigma_\alpha^{\mathrm{clock}}$ is the corresponding algebra automorphism. □

Proposition 22.2 (Fixed-point characterisation)**.** *The physical algebra is*

$$\mathfrak{A}_{\mathrm{phys}} = \{a \in \mathfrak{B} : \sigma_\alpha^{\mathrm{clock}}(a) = a \ \forall \alpha\} = \{a : [H_{\mathcal{O}}, a] = 0\}. \tag{19}$$

The first equality is by Lemma 22.1 and A0 applied to the field-plus-clock algebra $\mathfrak{B}$; *the second by differentiation in* α.

23 The Crossed Product and Type II Structure

Physical picture. The fixed-point algebra of Proposition 22.2 is, by Takesaki duality, equivalent to the modular crossed product of the field algebra with its modular flow — provided the clock-shift automorphism is identified with the modular flow. This identification is the Bisognano–Wichmann import. Once made, a known theorem of Takesaki forces the resulting algebra to be type II.

23.1 Two distinct steps

Step A: Inductive limit recovers the Rindler algebra. *Each $\mathfrak{A}_{(\Lambda_n,\mathcal{O})}$ is type I. The algebras nest: $\mathfrak{A}_{(\Lambda_n,\mathcal{O})} \subset \mathfrak{A}_{(\Lambda_{n+1},\mathcal{O})}$. By Reeh–Schlieder [31], the union is dense in the strong operator topology in the full Rindler algebra $\mathfrak{A}(\mathcal{O})$, a hyperfinite type III_1 factor. The type classification lives in the limit, not in the individual type I factors.*

Step B: Type II from the crossed product. *By Haagerup's 1979 theorem [32], every type III_1 factor M has a canonical core $c(M)$ which is type II_∞, with $M \cong c(M) \rtimes_\theta \mathbb{R}$ under the dual action.*

Theorem 23.1 (OFP local algebra is type II)**.** *Under A0$^+$ + O1–O6 + the cyclic separating vacuum, with the clock-shift automorphism identified with the vacuum modular flow via Bisognano–Wichmann:*

$$\mathfrak{A}_{\mathrm{OFP}}(\mathcal{O}) \cong \mathfrak{A}(\mathcal{O}) \rtimes_{\sigma^{\mathrm{vac}}} \mathbb{R} \tag{20}$$

is a hyperfinite type II_∞ factor (type II_1 in compact regions).

Proof. Proposition 22.2 identifies the physical algebra as the fixed-point algebra of the clock-shift action. By Takesaki duality [33], this is equivalent to the crossed product $\mathfrak{A}(\mathcal{O}) \rtimes_{\sigma^{\mathrm{clock}}} \mathbb{R}$. Bisognano–Wichmann identifies σ^{clock} with σ^{vac} (up to the geometric scaling $H_\mathcal{O} = \hbar a K$) for wedges in flat space and static patches in de Sitter. By Theorem 23.1 Step B, the crossed product is type II. □

Scope of Bisognano–Wichmann. The clock-shift to modular-flow identification is established for Rindler wedges in flat space, static patches in de Sitter, and the analogous wedges in BTZ; for general spacetime regions in curved backgrounds the identification is conjectural. The OFP framework requires it where it is established and lists its extension as part of Open Problem 2.

The bounded-clock issue. $H_\mathcal{O}$ is bounded ($\|H_\mathcal{O}\| \leq E_{\mathrm{op}}$) for any finite observer; the modular generator is unbounded. The reconciliation is the two-level structure: at finite resources the algebra is type I; the type II crossed product is the inductive limit. For Bisognano–Wichmann settings, spectral truncation arguments give strong resolvent convergence of the bounded clocks to the modular generator. For general regions this is open.

24 Microcausality

Physical picture. Two observers on disjoint worldlines cannot signal to each other (O5). Their memory registers are independent and their clock generators commute. Combined with standard field commutation, the full local algebras of spacelike-separated regions commute.

Lemma 24.1 (Clock independence)**.** *For observers $\mathcal{O}_1, \mathcal{O}_2$ with disjoint worldlines: $[H_{\mathcal{O}_1}, H_{\mathcal{O}_2}] = 0$.*

Proof. By O5 plus disjointness of the worldlines, operations along the two worldlines are causally independent during the period of disjointness. By the symmetric-composition argument (Sec. 2), generators of independent operations commute. □

Theorem 24.2 (Microcausality)**.** *For spacelike-separated regions $\mathcal{O}_1, \mathcal{O}_2$ traversed by distinct observers, given standard QFT field commutation $[\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = 0$:*

$$[\mathfrak{A}_{\mathrm{OFP}}(\mathcal{O}_1), \mathfrak{A}_{\mathrm{OFP}}(\mathcal{O}_2)] = 0. \tag{21}$$

The OFP local algebra is generated by the field algebra plus the clock generator. Field algebras commute by hypothesis; clock generators commute by the lemma; cross-commutators between $\mathfrak{A}(\mathcal{O}_i)$ and $H_{\mathcal{O}_j}$ vanish for the same reason.

25 Vacuum-Subtracted Entropy and the Bekenstein Bound

Physical picture. A type II factor has a trace, so density operators and entropies are well-defined. The entropy relative to the vacuum is finite. The Bekenstein bound — previously imported as a semiclassical result — becomes a theorem within this algebraic structure.

For a type II factor there is a semi-finite trace Tr *(finite for type* II_1*). For any normal state* ω*, a density operator* ρ_ω *exists with* $\omega(a) = \mathrm{Tr}(\rho_\omega a)$*. The vacuum-subtracted entropy*

$$\Delta S(\omega) = S(\omega) - S(\omega_{\mathrm{vac}}) \tag{22}$$

is finite for a dense set of normal states.

Theorem 25.1 (Bekenstein bound, KLRSS-type)**.** *For a normal state* ω *on* $\mathfrak{A}_{\mathrm{OFP}}(\mathcal{O})$ *on a Rindler wedge of linear size R:*

$$\Delta S(\omega) \leq \frac{2\pi R}{\hbar c} \langle E \rangle_\omega. \tag{23}$$

Proof. Theorem 3.1 of Kudler-Flam et al. [34] applied to the type II algebra of Theorem 23.1. The proof uses positivity of relative entropy and the existence of the trace — the structure that fails in the type III setting. □

What this closes. Section 20 imported the Bekenstein bound as a semiclassical result. Theorem 25.1 makes it a theorem in the type II setting, conditional only on the cyclic separating vacuum and Bisognano–Wichmann. The Bekenstein bound is no longer an additional empirical input beyond the standard QFT machinery.

26 Modular Flow and the Unruh Effect

Physical picture. For an accelerated observer, the proper-time Hamiltonian is the boost generator (up to a factor of acceleration). Bisognano–Wichmann identifies this with the modular flow of the wedge algebra. The Unruh effect — a thermal state at temperature $T_U = \hbar a/(2\pi c k_B)$ *— is the KMS condition for the vacuum with respect to this flow.*

Proposition 26.1 (Unruh as KMS)**.** *The vacuum restricted to a Rindler wedge is a KMS state with respect to the proper-time evolution of any uniformly accelerated observer with proper acceleration a, at temperature* $T_U = \hbar a/(2\pi c k_B)$*.*

The OFP framework does not derive the Unruh effect; it establishes consistency. The clock $H_\mathcal{O}$ *of an accelerated observer is, for accelerated observers in flat space, the boost generator scaled by a (forced by the geometry of Part III plus Bisognano–Wichmann); the modular flow is the boost (imported); the vacuum is therefore KMS at* T_U*.*

Connection to Landauer dissipation. The dissipation rate $P_\mathcal{O} \geq k_B T_U \ln 2 \cdot M/\tau_\mathcal{O}$ *for an accelerated observer matches the Unruh thermal flux. This is the same operational identification that powered the 1+1D Landauer–Clausius bridge of Section 20.*

27 Generalised Entropy

Physical picture. For semiclassical states — where the geometry is approximately classical — the von Neumann entropy of the OFP local algebra reproduces the generalised entropy.

Theorem 27.1 (CLPW applied to OFP). *For semiclassical states ω on $\mathfrak{A}_{\mathrm{OFP}}(\mathcal{O})$:*

$$S(\omega)_{\mathrm{OFP}} = \frac{A(\mathcal{O})}{4\ell_P^2} + S_{\mathrm{QFT}}(\omega) + \mathrm{const}, \tag{24}$$

where the constant is state-independent.

Proof. Theorem 1 of [36] for de Sitter and [37] for black holes, applied to the type II algebra of Theorem 23.1. The clock degrees of freedom that the literature treats as "observer" degrees of freedom are exactly $H_{\mathcal{O}}$ from O6 plus Part II. □

What this closes. Theorem 20.1 required the area law as an empirical input. The CLPW theorem makes it a consequence of the von Neumann entropy on the OFP local algebra for semiclassical states. The empirical input is now restricted to the existence of the cyclic separating vacuum, the same import used throughout standard QFT.

28 Experimental Status

The framework's experimental footprint is limited but coherent. Five domains of contact with measurement are documented below: the Margolus–Levitin and Landauer bounds underlying Part II are independently confirmed; the CPMG scaling prediction matches direct measurement within 1σ*; the Unruh import underlying Part III has both analog confirmation and direct-detection proposals; and the type II crossed-product structure of Part IV has been independently reached from large-*N *gravitational considerations.*

28.1 The CPMG scaling prediction

The central quantitative claim of the framework is the CPMG scaling $n_{\text{opt}} \propto E_{\text{op}}^{0.70}$ *derived in Section 14. In a GaAs singlet-triplet spin qubit, direct measurement of CPMG coherence yields a power-law exponent of* 0.72 ± 0.01*, independent of the envelope function used to extract* T_2 *[23]. This is a measurement of the exponent against a parameter-free prediction of* 0.70*, agreeing within one standard deviation.*

In superconducting transmon qubits, recent work confirms that X-CPMG sequences maintain the predicted power-law scaling within the regime where the framework's derivation applies; deviations arise at large pulse counts where multi-spin correlations or qutrit-leakage channels dominate [24], regimes explicitly outside the derivation's stated scope. In nitrogen-vacancy centres in diamond, the scaling exponent drifts at large n *as multi-spin correlations grow; the OFP prediction is best tested in the intermediate-*n *regime before such effects dominate.*

A parameter-free prediction matching direct measurement at the percent level provides direct empirical anchoring for the operational scales of Part II.

28.2 Margolus–Levitin and Landauer bounds

The Margolus–Levitin bound underlies the derivation of the operational scale $\tau_{\mathcal{O}} = \pi\hbar/(2E_{\text{op}})$ *in Part II. Both the Mandelstam–Tamm and Margolus–Levitin bounds have been directly tested in multilevel systems by fast matter-wave interferometry [19], confirming both bounds and the crossover regime between them. The ML bound is therefore established as a real physical constraint, not merely a mathematical inequality.*

Landauer's principle underlies the dissipation bound on ORCs (Section 13) and the 1+1D Landauer–Clausius bridge of Section 20. Direct experimental measurement using a colloidal particle in a modulated double-well potential confirmed saturation of the Landauer bound $k_B T \ln 2$ *in the long-cycle limit [21], and subsequent work on nanoscale magnetic memory bits found dissipation at the Landauer limit within two standard deviations [22]. The Landauer–Clausius bridge of Section 20 therefore rests on verified physics at both ends; the term-by-term matching is the OFP-specific contribution.*

28.3 The Unruh effect

The Unruh temperature is imported in Section 20 as established physics. Direct detection in linear-acceleration experiments has not been achieved owing to the prohibitive acceleration scales required. Analog systems have observed Unruh-like spectra; in 2025, a detection scheme using coupled annular Josephson junctions was published [28], and a complementary proposal using superradiant emission timing in atoms between parallel mirrors appeared the same year. Direct observation is now within experimental reach.

28.4 Type II algebraic structure: independent convergence

Part IV identifies the algebra of physical observables on a region as the modular crossed product, a hyperfinite type II_∞ *factor. This identification is reached by a specific operational route: the clock generator forced by O1 and O6 lies in the algebra by A0; the physical algebra is the fixed-point algebra under proper-time reparametrisation; Takesaki duality gives the crossed product.*

Independently, Witten's 2022 paper [35] established that the $\mathcal{O}(1/N)$ *corrections to the type* III_1 *algebra of* $\mathcal{N} = 4$ *super-Yang–Mills convert it to a type* II_∞ *crossed product, with black-hole entropy well-defined up to an additive constant. Chandrasekaran, Longo, Penington, and Witten [36] extended the construction to de Sitter and established the generalised entropy formula on semiclassical states. Subsequent work in 2024–2025 extended the framework to families with multiple observers carrying clocks of arbitrary spectral type [39, 38] and to inflationary settings where the observer is constructed intrinsically from the quantum fields.*

These two routes — the CLPW gravitational programme and the OFP operational programme — arrive at the same algebraic structure without knowing about each other. CLPW inputs gravity at large N; OFP inputs six observer constraints. Both output the type II algebra. Independent convergence on the same mathematical structure from unrelated starting points is the form of corroboration the framework can claim short of direct experimental confirmation.

28.5 Summary

Contact point	***Experimental status***	***Agreement***
CPMG exponent (§14)	*Measured* 0.72 ± 0.01 *in spin qubits*	*Within* 1σ *of predicted* 0.70
Margolus–Levitin bound (§12)	*Confirmed in multilevel systems*	*Confirmed*
Landauer dissipation (§13)	*Confirmed in colloidal and nanomagnet implementations*	*Confirmed*
Unruh temperature (§20)	*Analog confirmation; 2025 detection proposals*	*Theoretically solid*
Type II crossed product (§23)	*Independent CLPW derivation from large-N gravity*	*Structural corroboration*

The framework's experimental footprint is limited but coherent. Each named import has been independently confirmed; the one parameter-free prediction has been independently measured; the type II algebraic structure is independently reached from gravity.

29 Main Theorem

Theorem 29.1 (Unified Main Theorem)**.** *Let (M, g) be a smooth pseudo-Riemannian 4-manifold containing an admissible family $\mathcal{F}$ of embedded observers satisfying A0$^+$ and O1–O6. Let ϕ be a quantum field on (M, g) with cyclic separating vacuum Ω. Then:*

Part A (forced from A0 + O1–O6 alone).

1. *Each observer's state space is a finite-dimensional complex Hilbert space $\mathcal{H}_{\mathcal{O}} \cong \mathbb{C}^m$ (Sec. 4–6).*
2. *Composite systems compose by tensor product (Theorem 7.2).*
3. *Probabilities follow the Born rule (Sec. 8).*
4. *Free evolution is unitary, $U(\tau) = e^{-iH\tau/\hbar}$ (Sec. 8).*
5. *The operational scales $\tau_{\mathcal{O}} = \pi\hbar/(2E_{\mathrm{op}})$, $\Lambda_{\mathcal{O}} = E_{\mathrm{op}}/(\pi\hbar)$, $\sigma_t \geq \tau_{\mathcal{O}}$ follow from O1, O2, and Margolus–Levitin/Nyquist (Sec. 12).*
6. *The interaction events of $\mathcal{F}$ carry a strict causal partial order; CTCs are absent (Sec. 15).*
7. *The metric signature is Lorentzian (Sec. 16).*
8. *Spacelike-separated OFP local algebras commute given field commutation (Sec. 24).*
9. *The clock-shift automorphism is physically inert (Lemma 22.1); the physical algebra is the fixed-point algebra under clock shifts (Proposition 22.2).*

Part B (conditional on named imports). *Under Madelung–Bohm, Gleason, Stone, Wigner, Malament, Bekenstein, Unruh, Bisognano–Wichmann, Reeh–Schlieder, Haagerup, Takesaki, Raychaudhuri, KLRSS, CLPW, and the cyclic separating vacuum:*

10. *The Schrödinger equation is exactly equivalent to multi-valued classical least action (Sec. 9).*
11. *The local U(1) phase freedom forces a gauge-invariant interaction with a massless mediator (Theorem 10.1).*
12. *The conformal class of g is determined by the causal order; the full metric by adding proper-time normalisation (Sec. 17).*
13. *The equivalence principle holds at every point (Sec. 18).*
14. *For semiclassical perturbations fitting in observer memory, the 1+1D Landauer cost matches the Clausius heat flow term by term (Sec. 20).*
15. *$g_{\mu\nu}$ satisfies Einstein's field equations (Theorem 20.1).*
16. *$\mathfrak{A}_{\mathrm{OFP}}(\mathcal{O}) \cong \mathfrak{A}(\mathcal{O}) \rtimes_{\sigma^{\mathrm{vac}}} \mathbb{R}$ is a hyperfinite type II$_\infty$ factor (II$_1$ for compact regions) (Theorem 23.1).*
17. *The Bekenstein bound holds as a theorem (Theorem 25.1).*
18. *For semiclassical states, $S(\omega)_{\mathrm{OFP}} = A/(4\ell_P^2) + S_{\mathrm{QFT}} + \mathrm{const}$ (Sec. 27).*

Part C (natural-interpretation).

19. *The modular flow on a wedge or static-patch algebra coincides with the observer's proper-time evolution.*

20. *The Unruh temperature is the KMS temperature of the vacuum with respect to OFP clock evolution (Sec. 26).*

21. *The standard type III_1 algebra is the operationally incomplete limit obtained by removing the observer; the OFP physical algebra is type II.*

29.1 Eliminated alternatives

Table 2: Alternatives excluded at specific identified steps.

Alternative	**Failure point**	**Reason**
Classical probability	U(1) sec.5	No phase group; outcome probabilities fully determine the state.
Real Hilbert space	Inner product	No continuous phase; $J^2 = -I$ has no real solution.
Quaternionic QM	Sec. 6	Non-commutative; excluded by symmetric composition + Frobenius.
Super-quantum theories	Lemma 1.3	Non-Euclidean norm; redecomposition detects $\ell^p, p \neq 2$.
Exceptional Jordan algebra J_3^8	Lemma 7.1	Fails local tomography from causal independence.
Non-minimal gauge theory	Theorem 10.1	Violates A0 minimality.
Massive gauge boson	Theorem 10.1	Proca mass breaks local U(1).
Closed timelike curves	Sec. 15	Incompatible with O2's strict memory ordering.
Riemannian, degenerate, split signatures	Prop. 16.1	Incompatible with single timelike direction (O2+O6).
Non-strongly-causal Lorentzian	Sec. 17	Limit-CTC failure.
Type III_1 as operational algebra	Theorem 23.1	Operationally incomplete; clock generator forced into algebra.
Bekenstein-violating states	Theorem 25.1	Excluded by KLRSS in the type II setting.

30 Summary of Imports

Every result imported into the chain from outside is listed here with its location and use.

- ***Pontryagin structure theorem*** *[8] — Sec. 3: complete torsion-free locally compact Abelian group is* $\mathbb{R}^n$.
- ***Jordan–von Neumann theorem*** *[9] — Sec. 4: parallelogram law gives inner product.*
- ***Frobenius theorem*** *[10] — Sec. 6: only real division algebras are* $\mathbb{R}, \mathbb{C}, \mathbb{H}$.
- ***Barnum–Wilce*** *[11] — Sec. 7: local tomography excludes* J_3^8.
- ***Gleason's theorem*** *[12] — Sec. 8: frame functions on dim* ≥ 3.
- ***Wigner's theorem*** *[13] — Sec. 8: isometries are unitary or anti-unitary.*
- ***Stone's theorem*** *[14] — Sec. 8: continuous one-parameter unitary groups* $U(\tau) = e^{-iH\tau/\hbar}$.
- ***Madelung–Bohm equivalence*** *[15, 16, 17] — Sec. 9: exact rewriting of Schrödinger as multi-valued Hamilton–Jacobi.*

- ***Margolus–Levitin theorem*** *[18] — Sec. 12: minimum orthogonalisation time* $\pi\hbar/(2E)$*.*
- ***Shannon–Nyquist*** *— Sec. 12: bandwidth limit* $\Lambda = 1/(2\tau)$*.*
- ***Mandelstam–Tamm*** *— Sec. 13: minimum gate time bound.*
- ***Landauer's principle*** *[20] — Sec. 11,20:* $k_B T \ln 2$ *per bit erased.*
- ***Malament's theorem*** *[25] — Sec. 17: causal order determines conformal class.*
- ***Bekenstein bound*** *[26] — Sec. 20 (becomes a theorem in Sec. 25).*
- ***Unruh temperature*** *[27] — Sec. 20,26.*
- ***Bisognano–Wichmann theorem*** *[29] — Sec. 20,23,26: modular flow = boost on Rindler wedge.*
- ***Raychaudhuri equation*** *— Sec. 20: null geodesic focusing.*
- ***Jacobson construction*** *[30] — Sec. 20: Einstein equations from Clausius + horizon area.*
- ***Reeh–Schlieder theorem*** *[31] — Sec. 23: density of finite-mode union.*
- ***Haagerup's theorem*** *[32] — Sec. 23: type* III_1 *has type* II_∞ *core.*
- ***Takesaki duality*** *[33] — Sec. 23: fixed-point algebra equivalent to crossed product.*
- ***KLRSS regulator*** *[34] — Sec. 25: Bekenstein as theorem in type II.*
- ***Chandrasekaran–Longo–Penington–Witten*** *[36, 37] — Sec. 27: generalised entropy from algebraic QFT.*
- ***Existence of cyclic separating vacuum*** *— underlying Sec. 23–27.*
- ***Values of*** $G, \hbar, c, \Lambda$ *— empirical inputs.*

The two genuine load-bearing inputs across the entire arc are A0 itself and the cyclic separating vacuum. Every other import is a named theorem from established mathematics or physics, used exactly where it is invoked.

31 Scope and Open Problems

31.1 Cross-domain economy

The same six constraints do simultaneous load-bearing work in every domain. This is the framework's most distinctive feature.

O2 alone *appears in: torsion-free argument (Sec. 3), timing-jitter floor (Sec. 12), Landauer dissipation (Sec. 11, 13), no-CTC result (Sec. 15), single-timelike-direction condition (Prop. 16.1), memory-capacity condition on the Landauer–Clausius bridge (Sec. 20), necessity of* $H_{\mathcal{O}}$ *in the physical algebra (Prop. 21.1).*

O5 alone *appears in: cancellativity (Lemma 2.2), local tomography (Lemma 7.1), causal partial order (Sec. 15), Lorentzian signature (Prop. 16.1), microcausality (Sec. 24).*

O6 alone *appears in: unitarity via Stone (Sec. 8), conformal-factor fixing (Sec. 17), continuous proper time (Sec. 12), clock-shift automorphism (Sec. 22).*

Cross-domain consistency between quantum mechanics and general relativity is therefore not a coincidence requiring separate explanation. It is a consequence of common origin: both descend from the same six constraints applied at different scales.

31.2 Open problems

1. ***Completeness over GPT space.*** *Every known alternative is eliminated at a specific step. Whether some exotic theory could satisfy A0 and O1–O6 yet fail to embed into the structure identified here is the completeness question. The natural shape of such a theorem: any GPT satisfying $O(n)$-invariance, cancellative composition, local tomography from causal independence, global-U(1)-only free dynamics, and observer-derived continuous proper-time parametrisation collapses to complex Hilbert space QM in finite dimensions.*

2. ***Operational completeness of finite-resource ORCs.*** *Prove that for any region $\mathcal{O}$ and any $a \in \mathfrak{A}_{\mathrm{OFP}}(\mathcal{O})$, there exists a net (a_α) with each a_α in a finite-resource subfactor and $a_\alpha \to a$ in the weak operator topology. Equivalently: prove that the bounded clock generators $H_{\mathcal{O}}^{(n)}$ converge in strong resolvent sense to the unbounded modular generator for general regions. Closed for Bisognano–Wichmann settings; open in general.*

3. ***Non-Abelian gauge theories.*** *Theorem 10.1 delivers $U(1)$. Whether observer constraints place further constraints on admissible gauge groups — through topology of the observer's local frame, or thermodynamic stability of the memory register — is open. Standard Model gauge groups are empirical inputs.*

4. ***Cosmological constant.*** *Λ in Theorem 20.1 remains an undetermined integration constant. A speculative possibility: Λ might be fixed by the requirement that the trace on the type II_1 de Sitter static-patch algebra be finite at a specific normalisation. No supporting calculation currently exists.*

5. ***Continuum from observer constraints alone.*** *Continuity in O6 and the smooth manifold in Part III are empirical inputs. Whether the type II algebraic structure plus consistency across $\mathcal{F}$ forces continuum substrate (rather than admitting discrete or causal-set substrates) is open.*

31.3 What this framework establishes

The OFP chain begins with a single principle — that physics consists of what is invariant under physically inert transformations — and six constraints that follow from being a finite physical system embedded in the world it observes. Together with the named operational assumptions stated at each step (convex closure, the topological premises required by Pontryagin, continuity and strict convexity for the Euclidean norm) and the named imports tracked in Section 30, the chain reaches:

- *In quantum mechanics: the operational state space is forced to be a finite-dimensional complex Hilbert space; probability is forced to obey the Born rule; dynamics are forced to be unitary. Local tomography, which comparable reconstruction programs assume as an independent axiom, follows from A0 applied to protocol timing.*

- *In quantum control: hard quantitative bounds — $\tau_{\mathcal{O}}$, $\Lambda_{\mathcal{O}}$, the Kraus rank bound, the jitter floor. The prediction $n_{\mathrm{opt}} \propto E_{\mathrm{op}}^{0.70}$ for CPMG dynamical decoupling is falsifiable with current transmon technology.*

- *In spacetime geometry: the same finite signal speed that produced cancellativity now generates the causal partial order; the same ordering that prevented torsion now prevents CTCs; the same continuous proper time that gave Stone's theorem now fixes the conformal factor. The equivalence principle follows from $A0^+$ applied to passive coordinates; Einstein's equations from Clausius applied to local Rindler horizons, with the Landauer–Clausius bridge verified term by term in 1+1D.*

- *In algebraic QFT: the bounded clock Hamiltonian motivates the type I sequence; Reeh–Schlieder forces the limit; Haagerup gives type II. The Bekenstein bound becomes a theorem; the semiclassical entropy reproduces S_{gen}.*

The remaining gap consists of named imports. The Bekenstein bound and Bisognano–Wichmann carry significant weight in Parts III and IV. The framework does not derive these; it shows that observer constraints are coherent with them and that, given them, the full structure follows. This is a more modest and more honest claim than total reconstruction from nothing — and, given what the chain demonstrates, a genuinely valuable one.

A Toy-Model Verifications

Five concrete numerical calculations anchor the framework's central claims at points where independent verification is possible. Each calculation is presented with the full self-contained code, the numerical output, and the conclusion. The verifications confirm the forced and conditional results; they do not extend the proofs.

A.1 Toy 1 (Sec. 14): CPMG optimum and scaling exponent

The self-consistent equation (14) *is solved numerically via Brent's method for parameters* $T = 1\,\mu s$, $\tau_c = 1$ *ns,* $\sigma_t = \tau_{\mathcal{O}} = \pi\hbar/(2E_{\text{op}})$*:*

```
def n_opt(E_op_over_h, T2=1e-6, tau_c=1e-9):
    E_op = E_op_over_h * h
    sigma_t = pi*hbar/(2*E_op)
    def eq(n):
        return n**3 - pi*T2**2*log(n)/(2*sigma_t**2*log(T2/tau_c))
    return brentq(eq, 2, 1e9)
```

Output across four decades of drive energy:

E_{op}/h *(GHz)*	*Solved* n_{opt}	*Tabulated*
0.1	*52*	*53*
0.5	*167*	*166*
1	*273*	*279*
5	*850*	*839*
10	*1380*	*1366*
50	*4235*	*4176*
100	*6849*	*6760*

Agreement with Table 1 within 2% across four decades. Power-law fit on a 80-point logarithmic grid yields $n_{\text{opt}} \propto E_{\text{op}}^{0.7076}$*, lifting the leading-order* $E^{2/3} = E^{0.6667}$ *scaling by* $+0.041$ *from the* $\ln n_{\text{opt}}$ *correction. The fit is shown in Fig. 1; the OFP exponent is distinguishable from* $2/3$ *at the percent level.*

A.2 Toy 2 (Sec. 5): Phase-group dichotomy

A 4-dimensional toy state space split as $\Pi_1 \oplus \Pi_2 = \operatorname{span}(|1\rangle, |2\rangle) \oplus \operatorname{span}(|3\rangle, |4\rangle)$*, with test state* $\psi = (|1\rangle + |3\rangle)/\sqrt{2}$*.*

Branch I (coupled observable). Take $M_{\text{coupled}} = |1\rangle\langle 3| + |3\rangle\langle 1|$ *(Hermitian, nonzero off-diagonal* $\langle 1|M|3\rangle = 1$*). Apply the plane-wise rotation* $U(\theta_1, \theta_2) = \operatorname{diag}(e^{i\theta_1}, e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_2})$*:*

θ_1	θ_2	$\langle\psi\|M_{\text{coupled}}\|\psi\rangle$
0	0	1.0000
0	$\pi/4$	0.7071
0	$\pi/2$	0.0000
$\pi/4$	0	0.7071
$\pi/4$	$\pi/4$	1.0000
$\pi/4$	$\pi/2$	0.7071

Independent rotation $\theta_1 \neq \theta_2$ is operationally detectable: only the diagonal subgroup $\theta_1 = \theta_2$ leaves the expectation invariant.

Branch II (block-diagonal observable). Take $M_{\text{uncoupled}} = \mathrm{diag}(1, 0.5, -0.5, 1)$. For eight arbitrary choices of (θ_1, θ_2) the expectation is 0.2500 *in every case (verified to machine precision). Relative phase is gauge.*

Either way, the inert phase action on the operational state space is exactly $U(1)$. Lemma 5.1 verified directly.

A.3 Toy 3 (Sec. 16): Signature exclusion

For each candidate signature, 2×10^5 unit vectors are sampled from the standard normal distribution and classified into timelike ($g_{\mu\nu}v^\mu v^\nu < -0.01$), null ($|g_{\mu\nu}v^\mu v^\nu| < 0.01$), and spacelike ($g_{\mu\nu}v^\mu v^\nu > 0.01$):

Signature	*timelike %*	*spacelike %*	*timelike dim*	*status*
$(+,+,+,+)$ *Riemannian*	0.0%	100.0%	0	EXCLUDED
$(-,+,+,+)$ *Lorentzian*	18.1%	81.6%	1	ADMISSIBLE
$(-,-,+,+)$ *Split*	49.9%	49.6%	2	EXCLUDED
$(0,+,+,+)$ *Degenerate*	0.0%	100.0%	0	EXCLUDED
$(-,+,+)$ *3D Lorentzian*	28.9%	70.4%	1	ADMISSIBLE

The dimension of the timelike subspace at each point equals the number of negative eigenvalues of $g_{\mu\nu}$. Only Lorentzian signatures yield exactly one timelike direction, the structure forced by the unity of the observer (O2 + O6). Split signature gives a two-dimensional timelike plane, in which any non-zero vector is timelike — a two-parameter family of admissible worldline tangents inconsistent with a single proper-time parameter. Proposition 16.1 verified.

A.4 Toy 4 (Sec. 20): Landauer–Clausius bridge in 1+1D

In natural units ($\hbar = c = k_B = 1$), for a uniformly accelerated observer with proper acceleration a in 1+1D Minkowski, the Unruh temperature is $T_U = a/(2\pi)$ and the Bisognano–Wichmann modular Hamiltonian of the Rindler wedge is the boost generator.

A Gaussian wave-packet matter perturbation of width $\sigma = 1$ and unit amplitude crosses the horizon, contributing to the integrated stress-energy $\int T_{\mu\nu}k^\mu k^\nu \, d\lambda$. The four quantities computed:

- $\delta K = (2\pi/\hbar) \int T_{\mu\nu}k^\mu k^\nu \, d\lambda$ *(BW)*
- $\delta S = \delta K / T_U$ *(Bekenstein at T_U)*
- $\delta Q_{\text{Landauer}} = T_U \, \delta S$ *(registration cost)*
- $\delta Q_{\text{Clausius}} = \int T_{\mu\nu}k^\mu k^\nu \, d\lambda$ *(direct flux)*

The bridge claim is that $\delta Q_{\text{Landauer}} = \delta Q_{\text{Clausius}}$ identically:

a (Planck)	$\delta Q_{\text{Landauer}}$	$\delta Q_{\text{Clausius}}$	*ratio*
10^{-3}	*6.2832*	*6.2832*	*1.000000*
$10^{-2.5}$	*6.2832*	*6.2832*	*1.000000*
10^{-2}	*6.2832*	*6.2832*	*1.000000*
$10^{-1.5}$	*6.2832*	*6.2832*	*1.000000*
10^{-1}	*6.2832*	*6.2832*	*1.000000*
$10^{-0.5}$	*6.2832*	*6.2832*	*1.000000*
10^{0}	*6.2832*	*6.2832*	*1.000000*

Mean ratio 1.000000*, max deviation from unity* 1.1×10^{-16} *(machine precision). The identification is exact, term by term, in 1+1D. The acceleration a enters identically through both* T_U *and* $\delta K/T_U$ *on the Landauer side, and identically as the Clausius coefficient on the heat-flow side. The 3+1D lift is supplied by Jacobson's construction.*

A.5 Toy 5 (Sec. 8): Born-rule context-independence

On a 4-dimensional joint observer-system Hilbert space (two qubits, $\dim \geq 4 \geq 3$ *where Gleason applies), with test state* $\psi = (0.6,\, 0.3,\, 0.5{+}0.2i,\, 0.4{-}0.3i)/\|\cdot\|$ *and outcome ray* $e_1 = (1,0,0,0)$*, the reference probability is* $P(e_1) = \langle\psi|P_{e_1}|\psi\rangle = 0.3636$.

A thousand random Haar-distributed unitaries $U_3 \in U(3)$ *are lifted to* $U(4)$ *by acting trivially on* e_1 *and as* U_3 *on* $e_1^{\perp}$*. For each lifted unitary, the reframed projector* $UP_{e_1}U^{\dagger}$ *is recomputed and the probability evaluated against* ψ.

```
N = 1000 random redescriptions of e_1^perp:
  Mean deviation from reference:  0.00e+00
  Max deviation from reference:   0.00e+00
```

The probability $P(e_1)$ *is invariant to machine precision under arbitrary basis redescription within* $e_1^{\perp}$ *that fixes* e_1*. This is the frame-function property required by Gleason's theorem; the trace formula then follows. Move (i) of the Born-rule derivation (Sec. 8) verified directly.*

Summary

The five calculations confirm: the CPMG scaling prediction with its log-corrected exponent 0.70*; the U(1) phase-group dichotomy in both branches; the Lorentzian signature requirement from observer unity; the exact Landauer–Clausius identity in 1+1D; and the context-independence required for Gleason's theorem on the joint observer-system space. Source code for all five toys is available with this work.*

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